Eigenvectors without Eigenvalues (2×2)

Solve: $\lambda \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$$\begin{cases} \lambda x = ax + by \\ \lambda y = cx + dy \end{cases}$$

$$\begin{cases} \lambda xy = axy + by^2 \\ \quad \shortparallel \\ \lambda xy = cx^2 + dxy \end{cases}$$

$$cx^2 + dxy = axy + by^2$$

$$cx^2 + (d-a)xy - by^2 = 0$$

Factor

$$(\alpha_1 x + \beta_1 y)(\alpha_2 x + \beta_2 y) = 0$$

Eigenvectors:

$\alpha_1 x + \beta_1 y = 0$ and $\alpha_2 x + \beta_2 y = 0$

$$\begin{bmatrix} \beta_1 \\ -\alpha_1 \end{bmatrix} \qquad \begin{bmatrix} \beta_2 \\ -\alpha_2 \end{bmatrix}$$

Quadratic Form?

$$cx^2 + (d-a)xy - by^2 = \begin{bmatrix} -y & x \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

EX $\begin{bmatrix} -2 & 3 \\ 4 & -6 \end{bmatrix}$

$$4x^2 - 4xy - 3y^2$$

$$(2x - 3y)(2x + y)$$

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

EX $\begin{bmatrix} 5 & 1 \\ -4 & 1 \end{bmatrix}$

$$-4x^2 - 4xy - y^2$$

$$-(2x + y)^2$$

$\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ — repeated eigenval.

Not always good for repeated eigenval

EX $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$$0x^2 + 0xy + 0y^2$$

2-D eigensp.

2D Repeated eigenval, nondegen. (diagonal)

EX $\begin{bmatrix} -1 & -1 \\ 5 & 3 \end{bmatrix}$

$$5x^2 + 4xy + y^2$$

$$((2+i)x + y)((2-i)x + y)$$

$$\begin{bmatrix} -1 \\ 2+i \end{bmatrix} \qquad \begin{bmatrix} -1 \\ 2-i \end{bmatrix}$$

Finding eigenvectors without computing eigenvalues.

$$\lambda \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{cases} \lambda x = ax + by \\ \lambda y = cx + dy \end{cases}$$

$$\begin{cases} \lambda = \dfrac{ax+by}{x} \\ \quad \parallel \\ \lambda = \dfrac{cx+dy}{y} \end{cases}$$

$$y(ax+by) = x(cx+dy)$$

$$0 = cx^2 + (d-a)xy - by^2$$

Factor

$$0 = (\alpha_1 x + \beta_1 y)(\alpha_2 x + \beta_2 y)$$

Eigenvectors: $\begin{bmatrix} \beta_1 \\ -\alpha_1 \end{bmatrix}$ & $\begin{bmatrix} \beta_2 \\ -\alpha_2 \end{bmatrix}$

Alt: $b^2 x + (d-a)xy - cy^2$

$$(\alpha_1 x + \beta_1 y)(\alpha_2 x + \beta_2 y)$$

$$\begin{bmatrix} \alpha_1 \\ \beta_1 \end{bmatrix} \qquad \begin{bmatrix} \alpha_2 \\ \beta_2 \end{bmatrix}$$

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} -y \\ x \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} b & -a \\ d & -c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Note: $cx^2 + (d-a)xy - by^2$

$$\parallel$$

$$\begin{bmatrix} -y & x \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

EX: $\begin{bmatrix} -2 & 3 \\ 4 & -6 \end{bmatrix}$

$$4x^2 - 4xy - 3y^2$$

$$(2x-3y)(2x+y)$$

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

EX: $\begin{bmatrix} 1 & -5 \\ 4 & -8 \end{bmatrix}$

$$4x^2 - 9xy + 5y^2$$

$$(4x-5y)(x-y)$$

$$\begin{bmatrix} 5 \\ 4 \end{bmatrix} \qquad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{cases} \lambda x = a_{11}x + a_{12}y + a_{13}z \\ \lambda y = a_{21}x + a_{22}y + a_{23}z \\ \lambda z = a_{31}x + a_{32}y + a_{33}z \end{cases}$$

$$\begin{cases} y(a_{11}x + a_{12}y + a_{13}z) = x(a_{21}x + a_{22}y + a_{23}z) \\ z(a_{11}x + a_{12}y + a_{13}z) = x(a_{31}x + a_{31}y + a_{33}z) \\ z(a_{21}x + a_{22}y + a_{23}z) = y(a_{31}x + a_{32}y + a_{33}z) \end{cases}$$

$$yz(a_{11}x + a_{12}y + a_{13}z)$$

$$\parallel$$

$$xz(a_{21}x + a_{22}y + a_{23}z)$$

$$\parallel$$

$$xy(a_{31}x + a_{32}y + a_{33}z)$$

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